STABILITY AND CONTROLLABILITY ANALYSIS ON LINEARIZED DYNAMIC SYSTEM EQUATION OF MOTION OF LSU 05-NG USING KALMAN RANK CONDITION METHOD

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ABSTRACT

This paper discusses the stability and controllability of the dynamic system of the LAPAN Surveillance UAV 05-NG (LSU 05-NG) aircraft equation. This analysis is important to determine the performance of aircraft when carrying out missions such as photography, surveillance, observation, and as a scientific platform to test communication-based on satellite. Before analyzing the dynamic system, first arranged equations of motion of the plane which include the force equation, moment equation, and kinematics equation. The equation of motion of the aircraft obtained by the equation of motion of the longitudinal and lateral dimensions. Each of these equations of motion will be linearized to obtain state-space conditions. In this state space, A, B, and C are linear matrices that will be obtained in the time domain. Stability analysis using eigenvalue method and controllability analysis using Kalman Rank Condition method. The results of the analysis of matrices A, B, and C show that the dynamic system in the LSU 05-NG motion equation is a stable system on the longitudinal dimension but the lateral dimension on the unstable spiral mode. As for the analysis of the control of both the longitudinal and lateral dimensions, the results show that the system is controlled.

Keywords: stability, controllability, Kalman rank condition, dynamical system.

1 Introduction

The development of unmanned aircraft in various countries is quite significant and the unmanned aircraft is widely applied to the needs of a country. Currently UAV (Unmanned Aerial Vehicle) or Drone aircraft have been used in various applications such as military industry, commercial cargo transportation, and mapping (Parmar & Acharya, 2015). In Indonesia, the development of UAV technology is also developing rapidly and its application is not inferior to other countries. Therefore, as a government agency engaged in the world of aeronautics, Pusat Teknologi Penerbangan continues to conduct research and innovation. One focus of work carried out is the development or development of unmanned aircraft. Until now, there are several variants of UAV owned. From the smallest to the biggest. One of the drones developed was the LAPAN Surveillance UAV 05 Next Generation (LSU 05-NG) aircraft.

LAPAN Surveillance UAV 05 Next Generation (LSU-05 NG) is an unmanned aircraft capable of accommodating large loads (maximum 30 kg) that has missions for research, observation, patrol, surveillance, and SAR activities. The main mission of this aircraft is to support
aerial photography activities by carrying loads in the form of optical devices. In the future, this aircraft will also be used as a scientific platform to test the satellite-based communication system developed by the LAPAN Pustekbang and can also be used for border surveillance (Rizaldi, 2019).

Before carrying out this mission, it is necessary to analyze the stability, control, and observability of the state space matrices. This simulation analysis is carried out to ensure that the LSU 05-NG is stable and can be controlled properly when there is interference. Previous research on stability analysis on UAVs has been conducted by (Purwanto, 2012). The analysis carried out is a dynamic stability analysis on the UAV. The results obtained that the UAV has dynamically stable properties in the longitudinal and lateral dimensions. In addition, an analysis of static stability has also been carried out by (Sugandi et al., 2018). The study was conducted on a UAV with a tandem model or commonly called a tandem wing UAV. Research related to stability is also widely carried out as conducted by (Boschetti & Cárdenas, 2012), (Boschetti et al., 2010) and (Cardenas et al., 2004). On the other hand, research about controllability and observability has been carried out. (Younus & Ur Rahman, 2014) conducted research on control and accuracy to obtain solutions from dynamic systems on voltera type nonlinear matrices. Also conducted by (Tian et al., 2019) research related to control and observation. Research conducted on a linear dynamic system on a multi-agent system.

In this study an analysis of the A, B, C, and D lines that have been linearized in the state space state. The purpose of this analysis is to find out the stability and controllability of the dynamic system that is formed from the equation of the linearized motion of the aircraft.

2 Methodology

2.1 Stability

In designing an aircraft, the stability analysis is very important to know the aircraft’s ability to carry out the mission and the aircraft’s attitude when there is interference. Stability analysis on aircraft includes static and dynamic stability analysis. Static stability of an aircraft is generally the first type of stability evaluated by a designer. The static stability criteria for the three aircraft rotation modes (pitch, roll, and yaw) must be considered (Yechout, 2003). In addition, it needs to be evaluated for aircraft dynamic stability. The dynamic stability of the aircraft focuses on the time domain when the aircraft is moving and gets outside interference when in a state of balance or trim condition. An analysis of the aircraft’s dynamic stability needs to be carried out to determine the aircraft’s handling quality and features designed for the aircraft to run well or not when carrying out the mission. The basis for knowing this dynamic stability, the first thing to do is to arrange a plane motion differential equation or equation of motion (EOM). Then from the results of EOM compilation is carried out linearization and will get a matrices A, B, and C in the state space. The matrices will find the roots of their characteristics which will be used to determine stability.

Theorem (1): Given a system of differential equations

\[ \dot{x} = Ax \]  

(2-1)

where A is matrices of size $n \times n$ and has an eigenvalue $\lambda_1, \lambda_2, \ldots, \lambda_i$ with $i \leq n$. The differential equation system is said to be stable at $x = 0$ if and only if the characteristic roots of matrices A in the real domain are negative or $Re\lambda_i < 0$. (George, 2015).
2.2 Controllability

Given a system of linear differential equations in the time domain

\[ \dot{x}(t) = Ax(t) + Bu(t), \quad x(t_0) = x_0 \]  
\[ y(t) = Cx(t) + Du(t) \]  

(2-2) (2-3)

where matrices \( A, B, C \) dan \( D \) have size \( n \times n, n \times r, m \times n \), dan \( n \times r \).

Theorem (2) Kalman rank condition: The differential equation system in Eq. (2-2) dan (2-3) is said to be controlled at intervals \( [t_0, t_1] \) if and only if the control matrices

\[
[B \ AB \ ... \ A^{n-1}B]
\]

(2-4)

with size \( n \times nm \) has rank as much as \( n \). (Davis et al., 2009)

Theorem 2 shows that the system is said to be controllable output if with the unconstrained control vector \( u(t) \). Theorem 2 shows that the system is said to be controllable output if with the unconstrained control vector \( x(t_0) \) to the condition \( x(t_1) \) in interval \( t_0 \leq t \leq t_1 \).

Based on the understanding of Theorem 2-2 it is important to know that any initial and final state consisting of \( n \) components and if all components of the initial state can be controlled to \( n \) components that correspond to the final state, then the system can be controlled.

Whereas with the existence of an unrestricted \( u(t) \) controller, nothing is required except to transfer just any initial state that is given to any desired final state at a finite time interval (Subiono, 2013)

2.3 Equation of Motion

Airplanes can move on the X, Y, and Z axes. Airplane movements include rolling, pitching, and yawing. Rolling is a rolling motion made by aircraft on the longitudinal axis caused by the aileron control plane. Pitching is a nodding motion down and up on the lateral axis due to the elevator control plane. While yawing is a turning motion on the horizontal body caused by rudder control. LSU 05-NG has flaps for increase lift when taking off. In general, the anatomy of LSU 05-NG is given at Figure 2-1.

To get the equation of motion aircraft, used first-principle modelling (Adiprawita et al., 2008). This approach is used by involving the basic equations of mechanics and aerodynamics. This reduction involves a decrease in the force equation, the flight kinematics equation and the moment equation.

Based on Figure 2-2, the equation of force on the plane at coordinates \( X, Y \) and \( Z \) is the sum of the forces that occur on the plane which include gravity, thrust, and aerodynamic forces (Luckner, 2007). Mathematically, the force equation can be written as follows
\[ m(\ddot{u} + qw - rv) = -mg \sin \theta + F_{Ax} + F_{Ix} \]  
\[ m(\dot{v} + ru - pw) = mg \sin \phi \cos \theta + F_{Ay} + F_{Iy} \]  
\[ m(\dot{w} + pv - qu) = mg \cos \phi \cos \theta + F_{Az} + F_{Iz} \]  
\[ p\dot{l}_x + qr(l_x - l_y) - (pq + r)l_{xz} = L_A + L_I \]  
\[ q\dot{l}_y - pr(l_z - l_y) + (p^2 - r^2)l_{yz} = M_A + M_I \]  
\[ r\dot{l}_z + pq(l_y - l_x) + (qr - p)l_{xz} = N_A + N_I \]

The moment equation in the aerospace vehicle is often called the rotational equation and mathematically the moment equation can be written as follows (Ulu s & Ikbal, 2019)

\[ \rho \dot{l}_x + q(r(l_x - l_y) - (pq + r)l_{xz}) = L_A + L_I \]  
\[ q \dot{l}_y - p(r(l_z - l_y) + (p^2 - r^2)l_{yz}) = M_A + M_I \]  
\[ r \dot{l}_z + p(q(l_y - l_x) + (qr - p)l_{xz}) = N_A + N_I \]

To analyze an aircraft’s dynamic response, rotational angle motion is used on an aircraft that is \((\phi \ \theta \ \psi)\) which respectively states roll, pitch, and yaw (Ahmed et al., 2015). The relationship between angular velocity and rotation angle is expressed in the following mathematical form

\[ \dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta \]  
\[ \dot{\theta} = q \cos \phi - r \sin \phi \]  
\[ \dot{\psi} = q \sin \phi \sec \theta + r \cos \phi \sec \theta \]  

### 2.4 State Space Representations of the UAV Model

In this section, the equation of motion in the longitudinal and lateral dimensions is obtained from Eq. (2-5) to (2-13). Then those equation is linearized. Those longitudinal and lateral equations are obtained in the state space. Equations in the form of state space are needed to develop the transfer function for each state variable and control input. For longitudinal equations in state space conditions are given as follows

\[ \begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} A \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} B \end{pmatrix} \begin{pmatrix} \delta_e \\ \delta_r \end{pmatrix} \]  

with

\[ (A) = \begin{pmatrix} X_u & X_w & X_q & -g \cos \theta \\ Z_u & Z_w & Z_q & -g \sin \theta \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]

\[ (B) = \begin{pmatrix} X_{\delta_e} & X_{\delta_r} \\ Z_{\delta_e} & 0 \\ M_{\delta_e} & 0 \\ 0 & 0 \end{pmatrix} \]

The lateral motion equation on the plane involves the rolling moment, the yawing moment and the side force of the equation of motion. The lateral motion equation in the state space state is given as follows (Akyazi et al., 2013)

\[ \begin{pmatrix} \dot{\psi} \\ \dot{p} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} A_1 \end{pmatrix} \begin{pmatrix} \psi \\ p \\ r \end{pmatrix} + \begin{pmatrix} B_1 \end{pmatrix} \begin{pmatrix} \delta_d \\ \delta_r \end{pmatrix} \]  

with

\[ (A_1) = \begin{pmatrix} Y_v & Y_p & Y_r & -g \cos \theta \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]

\[ (B_1) = \begin{pmatrix} Y_{\delta_d} & Y_{\delta_r} \\ L_{\delta_d} & L_{\delta_r} \\ N_{\delta_d} & N_{\delta_r} \\ 0 & 0 \end{pmatrix} \]

### 3 Result and Discussion

Stability and control analysis is important to do to find out the flight performance of the aircraft when carrying out the mission. This analysis was carried out on the LSU 05-NG aircraft assuming when flying a cruise mission on a cruise with a speed of 30 m / s at an altitude of 1000ft. But before an analysis is carried out to obtain a dynamic system in the state of space, a simulation is
carried out to obtain aerodynamic stability derivatives. When conducting a simulation several variables to be considered as presented in Table 3-1.

Table 3-1: Variables that are considered during simulation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>1.222 kg/m$^3$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>1.463e-05 m$^2$/s</td>
</tr>
<tr>
<td>CG on X direction</td>
<td>1.515 m</td>
</tr>
<tr>
<td>CG on Y direction</td>
<td>0.000 m</td>
</tr>
<tr>
<td>CG on Z direction</td>
<td>0.047 m</td>
</tr>
<tr>
<td>Inertia on X</td>
<td>40.06 kg m$^2$</td>
</tr>
<tr>
<td>Inertia on Y</td>
<td>63.58 kg m$^2$</td>
</tr>
<tr>
<td>Inertia on Z</td>
<td>99.79 kg m$^2$</td>
</tr>
<tr>
<td>Inertia on XZ</td>
<td>4.28 kg m$^2$</td>
</tr>
<tr>
<td>Mass of aircraft</td>
<td>76.77 kg</td>
</tr>
</tbody>
</table>

The variables in Table 3-1 are entered into the XFLR5 software. In this XFLR5 software, the model of the aircraft was made first. The output of the aircraft model using XFLR5 is given at Figure 2-1: Model LSU 05-NG in XFLR5-1.

The results of this simulation provide stability derivative values which will be used as the basis for calculating the elements in matrices A, B, and C. The derivative of aerodynamic stability in question includes the stability derivatives used for longitudinal and lateral motion. The assumption of this simulation is in a state of the cruise where there is no rotation in the control plane or the control plane on the plane.

Derivatives of aerodynamic stability for aircraft motion on the longitudinal dimension are given as follows:

Table 3-2: Stability derivatives of longitudinal axis

<table>
<thead>
<tr>
<th>Stability derivatives of longitudinal</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{xu}$</td>
<td>-0.0012921</td>
</tr>
<tr>
<td>$C_{xa}$</td>
<td>0.11819</td>
</tr>
<tr>
<td>$C_{zu}$</td>
<td>-0.00069528</td>
</tr>
<tr>
<td>$C_{La}$</td>
<td>5.3799</td>
</tr>
<tr>
<td>$C_{Lq}$</td>
<td>11.331</td>
</tr>
<tr>
<td>$C_{mu}$</td>
<td>0.035372</td>
</tr>
<tr>
<td>$C_{ma}$</td>
<td>-4.4388</td>
</tr>
<tr>
<td>$C_{mq}$</td>
<td>-21.655</td>
</tr>
</tbody>
</table>

While the simulation results of aerodynamic stability derivatives for lateral motion are given in Table 3-3.

Table 3-3: Stability derivatives of lateral axis

<table>
<thead>
<tr>
<th>Stability derivatives of lateral</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{yv}$</td>
<td>-0.25652</td>
</tr>
<tr>
<td>$C_{yp}$</td>
<td>0.015134</td>
</tr>
<tr>
<td>$C_{yv}$</td>
<td>0.21229</td>
</tr>
<tr>
<td>$C_{tv}$</td>
<td>-0.012482</td>
</tr>
<tr>
<td>$C_{tp}$</td>
<td>-0.53234</td>
</tr>
<tr>
<td>$C_{tr}$</td>
<td>0.062209</td>
</tr>
<tr>
<td>$C_{nv}$</td>
<td>0.11286</td>
</tr>
<tr>
<td>$C_{np}$</td>
<td>-0.017257</td>
</tr>
<tr>
<td>$C_{nr}$</td>
<td>-0.093463</td>
</tr>
</tbody>
</table>

3.1 Analysis of Stability and Control on the Longitudinal Matrices
Based on the data in Table 3-2, obtained a matrices A and B in the longitudinal motion as follows

\[
A = \begin{pmatrix}
-0.00271615 & 0.248462 & 0 & -9.81 \\
-0.257616 & -11.3097 & 68.9497 & 0 \\
0.0576336 & -7.23232 & -11.3237 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
1.959083 \\
-73.99448 \\
-188.4752 \\
0
\end{pmatrix}
\]

Then to find the stability of the dynamic system, eigenvalues and pole locations will be sought in the imaginary domain plane. By using the determinant formula the values of the roots of matrices A are obtained as follows Table 3-4:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_1)</td>
<td>-11.31719 - 22.33163i</td>
</tr>
<tr>
<td>(\lambda_2)</td>
<td>-11.31719 + 22.33163i</td>
</tr>
<tr>
<td>(\lambda_3)</td>
<td>-0.00089 - 0.19840i</td>
</tr>
<tr>
<td>(\lambda_4)</td>
<td>-0.00089 + 0.19840i</td>
</tr>
</tbody>
</table>

Because the eigenvalues of matrices A are negative or less than zero, based on Theorem 1 (George, 2015) that the dynamic system composed of decreasing the motion of the LSU 05-NG aircraft is stable. In addition to analyzing the stability can be seen in Figure 3-2. In Figure 3-2, it appears that all characteristic roots are located to the left of the imaginary axis. This means that the LSU 05-NG is dynamically stable on the longitudinal dimension. On the other hand, it is necessary to do a phugoid mode analysis. Phugoid mode is a mode where a deviation occurs so that sinusoidal motion occurs at low frequencies. This needs to be analyzed also to determine the stability of the aircraft when there is interference. The phugoid mode analysis results are based on calculations using the following formula

\[
\lambda_{phugoid} = \frac{Z_u X_q - X_u Z_q}{2Z_q} \pm \sqrt{\left(\frac{Z_u X_q - X_u Z_q}{2Z_q}\right)^2 + \frac{g Z_u}{Z_q}} \quad (3-1)
\]

Based on the data in Table 3-2 and Eq. (3-1) the following characteristics for phugoid mode are obtained.

Table 3-5: Value of phugoid mode output characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_{phugoid})</td>
<td>-0.001358 ±0.16323i</td>
</tr>
<tr>
<td>Undamped Natural Frequency</td>
<td>0.163236Hz</td>
</tr>
<tr>
<td>Damped Natural Frequency</td>
<td>0.16323Hz</td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>0.008319</td>
</tr>
<tr>
<td>Time Period</td>
<td>37.86068737</td>
</tr>
</tbody>
</table>

In addition to the phugoid mode, a short period analysis is also performed. Where this short period is the oscillation motion in a shorter time than the phugoid mode. The analysis results for the short period are given in Table 3-6.

It can be seen in Figure 3-3 and Figure 3-4 the simulation results for phugoid mode and short periods of motion equations on the longitudinal dimension. Based on Figure 3-3 and Figure 3-4 there are differences. For Figure 3-3, because the damping ratio is very small, it takes a long time to return to a stable state. Whereas in Figure 3-6, because the damping ratio is quite large, the time to stabilize is quite fast.
Based on the analysis of the phugoid mode and short period, because the negative value is negative and based on the location of the pole as shown in Figure 3-2, the dynamic system composed of decreasing equations of motion at LSU 05 – NG is stable.

The next step is to analyze the controllability of the LSU 05-NG dynamic system using the Kalman rank condition. As the definition in the previous chapter, it will be proven that the linear system is said to be controlled if for any arbitrary condition \( x(0) \) there is an unlimited number of \( u(t) \) to transfer the state of \( x(0) \) to any final state \( x(t_1) \) in interval \( t_0 \leq t \leq t_1 \).

The solution of the system in Eq. (3-2) is as follows

\[
x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau
\]  

(3-2)

where \( A \) and \( B \) are linearized matrices for longitudinal motion. Assuming the dynamic system in Eq. (2-2) is a controlled system, there is a controller \( u(t) \) so that any \( x(0) \) can be transferred to \( x(t_1) \). By choosing \( t = t_1 \) Eq. (3-2) becomes

\[
x(t_1) = e^{At_1}x(0) + \int_0^{t_1} e^{A(t_1-\tau)}Bu(\tau)d\tau
\]  

(3-3)

To guarantee the existence of the controller \( u(t) \) which transfers \( x(0) \) to \( x(t_1) \) within a finite time a non-singular matrix is defined

\[
W(0, t_1) = \int_0^{t_1} e^{-At}BB^Te^{-A^T\tau}d\tau
\]  

(3-4)

where \( B^T \) is the transpose of \( B \) matrices. Define the \( u(t) \) is controller matrices as follows

\[
u(t) = -B^T e^{-A^T\tau}W^{-1}(0, t_1)[x(\tau) - e^{-At_1}x_1]
\]  

(3-5)

where \( W^{-1}(0, t_1) \) is invers matrices \( W(0, t_1) \).

By substituting Eq. (3-5) into Eq. (3-3), \( x(t_1) = x_1 \) is obtained. Its means that there is a controller \( u(t) \) so the dynamics system can be controlled. After that the rank condition of the matrices \( [B \ AB \ ... \ A^{n-1}B] \) will be evaluated. Because the size matrix \( A \) is 4\times4 then the value \( n = 4 \) the control matrix for the dynamic system in Eq. (2-2) is given as follows

\[
[B \ AB \ A^2B \ A^3B]
\]  

(3-6)

Based on matrices \( A \) and \( B \) generated in longitudinal motion, then Eq. (3-6) becomes

\[
M_{AB} = 10^6 \begin{bmatrix} 0 & 0 & -0.0012 & 0.0537 \\ -0.0001 & -0.0122 & 0.3216 & 0.3423 \\ -0.0002 & 0.0027 & 0.0577 & -2.9793 \\ 0 & -0.0002 & 0.0027 & 0.0577 \end{bmatrix}
\]

Because the determinant of matrices \( M_{AB} \neq 0 \), so matrices \( M_{AB} \) has rank equal to 4. Based on Theorem 2, the dynamic system that is composed of a linear equation of linear motion has controlled properties.
Figure 3-2: Pole position on the longitudinal dimension

Figure 3-3: Longitudinal dimension simulation results in phugoid mode

Figure 3-4: Longitudinal dimension simulation results in a short period
3.2 Analysis of Stability and Control in Lateral Dimension

Analysis of stability and controllability in the lateral dimension is done by calculating the values of the matrix elements A and B in the Eq. (2-15). Based on the data in Table 3-3: Derivative stability of lateral motion, the linear matrix A and B are obtained as follows:

\[
A = \begin{pmatrix}
-0.539251 & 0.0874892 & -75.3672 & 9.81 \\
-0.0240078 & -32.2421 & 3.18616 & 0 \\
1.01546 & -3.68163 & -1.99666 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
-20.83547 \\
-16.8183 \\
37.95013 \\
0
\end{pmatrix}
\]

By using eigenvalue method of matrices A, obtained the characteristic values of matrix A are as follows:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_1)</td>
<td>-31.88672+0.00000i</td>
</tr>
<tr>
<td>(\lambda_2)</td>
<td>-1.45186-8.76579i</td>
</tr>
<tr>
<td>(\lambda_3)</td>
<td>-1.45186+8.76579i</td>
</tr>
<tr>
<td>(\lambda_4)</td>
<td>0.01242+0.00000i</td>
</tr>
</tbody>
</table>

There are 3 modes if analyzed based on the root characteristics of matrix A, namely dutch roll, spiral, and roll subsidence. Three modes can be seen in Figure 3-5. Based on Table 3-7 and Figure 3-5 the spiral mode is positive and the pole is located to the right of the imaginary axis. This indicates that in spiral mode the plane’s motion is unstable. However, for dutch roll and roll subsidence, it is stable. If the longitudinal motion is analyzed for two modes, namely the short period and phugoid mode, then the lateral movement also analyzes the dutch roll mode. Dutch roll is motion instability caused by disturbance resulting in a combination of yawing and rolling motion. Simulation results for lateral motion in dutch roll mode are given in Figure 3-6.

Based on Figure 3-6, the yaw rate (r) has a peak value of 5.322272 at 0.15 seconds and returns to the settling time at 7.43 seconds. Roll rate (p) has a peak value at 0.2 seconds of 0.495407 degrees / s and returns stable at 6 seconds of the graph to zero. The angle of roll on the graph starts from -0.06867 in the initial seconds and has a peak value of 0.042045 degrees at the second to 0.39 and continues to oscillate until stable again at 4.49 seconds.

![Figure 3-5: Graph of a mode of lateral motion](image)
Next, the controllability of the dynamic system on the lateral dimension will be analyzed. The steps for analysis of controllability in the lateral dimension are the same as the analysis of controllability in the longitudinal dimension. The first thing to do is to prove the existence of the controller $u(t)$ so that any $x(0)$ can be transferred into $x(t_1)$. Therefore defined $u(t)$ as a controller as in Eq. (3-5). Based on the analysis done previously on the longitudinal dimension, the controller $u(t)$ can be guaranteed so that any $x(0)$ can be transferred to $x(t_1)$. When it can be guaranteed that there is a controller $u(t)$ will be analyzed matrice $N_{AB}$.

With the input matrices A and B on the lateral dimension obtained

$$N_{AB} = 10^5 \begin{bmatrix} -0.0002 & -0.0285 & 0.0407 & 3.9947 \\ -0.0002 & 0.0066 & -0.2144 & 6.7443 \\ 0.0004 & -0.0004 & -0.0527 & 0.9359 \\ 0 & -0.0002 & 0.0066 & -0.2144 \end{bmatrix}$$

Determinant of matrices $N_{AB}$ not equal to $n = 4$. Because the rank of matrices $N_{AB}$ is equal to $n$, then matrix $N_{AB}$ can be controlled in other words, dynamic systems of the lateral dimension are controlled. Simulations performed both on the longitudinal and lateral dimensions, are simulated by calculating the load carried by the LSU 05-NG when carrying out the mission. Based on simulations using XFLR5, when carrying out missions carrying 30 kg loads, the LSU 05-NG remains stable.

4 Conclusion

Analysis of the stability and control of the dynamic system in longitudinal and lateral motion shows that the dynamic system in the longitudinal dimension is stable and can be controlled by using eigenvalue method and Kalman Rank Condition Method. This is because matrix A has a negative characteristic value and the location of the pole in the s-plane is to the left of the imaginary axis. On the other hand, because the control matrix $M_{AB}$ has the same rank as $n$, the dynamic system in longitudinal motion can be controlled or controlled. Whereas in the lateral dimension, three eigenvalues or
negative characteristics are stable in the subsidence roll and dutch roll. However, in spiral mode, it is not stable due to positive eigenvalue. The $N_{AB}$ control matrix has the same rank as $n$, so it can be concluded that the dynamic system in the lateral dimension is controlled.

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AS developed the simulation; designed method, analyzed the results, and prepared the manuscript.

References
Tian, L., Guan, Y. & Wang, L. 2019. Controllability and observability of multi-agent systems with general linear dynamics under switching

